Gust Response of a Wing near the Ground through the Lifting Surface Theory

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The ground effect on the gust response of wings is investigated theoretically and experimentally. In the theoretical calculations the lifting surface theory is applied. The ground effect terms are added to the ordinary kernel function. The gust response functions are calculated for three rectangular wings of different aspect ratio and one delta wing. The augmentation of the response function by the ground effect appears as the wing approaches the ground. Furthermore, experiments of the ground effect on the wings are performed by wind-tunnel tests. Comparison between experimental values and theoretical results shows good agreement.

Nomenclature

a	= factor controlling number of spanwise integration points
$c(\eta),\bar{c}$	= local chord length and geometric mean chord length
d	= representative length
$E_q(x,y_l)$	= function in Eq. (18)
F_q,F_{hq}	= influence functions, Eq. (15)
F_1,F_2	= first and second term of the kernel function
k	= reduced frequency
I_l,K_l,L_l	= modified Bessel functions and Struve function
I_l',K_l',L_l'	= $\mathrm{d}I_l/\mathrm{d}z$, $\mathrm{d}K_l/\mathrm{d}z$, $\mathrm{d}L_l/\mathrm{d}z$
j	= $\sqrt{-1}$
$K(x_0,y_0,z;M,\omega)$	= kernel function
$\bar{K},\tilde{K}$	= Eq. (11)
l	= lift distribution on the wing
$\bar{l}$	= $l\cdot e^{-jkx}$
M	= Mach number
m	= number of the collocation points along span
N	= number of the collocation points along chord
P_q,P_q'	= modified influence function [Eq. (20)] and $\partial P_q/\partial y_l$
p	= integer denoting chordwise position
R_q	= regularized influence function, Eq. (19)
r	= integer denoting loading station
q	= integer denoting function ψ_q
r_l	= $(\omega/U)\sqrt{y_0^2+z^2}$
S,s	= area of wing planform and semispan of wing
$S(k),S'(k)$	= Sears function and modified Sears function, Eq. (25)
U	= freestream velocity
$\bar{w}$	= modified complex downwash
$\bar{w}_G$	= amplitude of gust
x,y,z	= coordinates on real wing
x',y',z'	= coordinates on imaginary wing
y_l	= y/s
x_l,x_t	= local leading and trailing ordinates
$x_{p\nu}$	= ordinate of collocation
x_0,y_0	= $x-\xi$ and $y-\eta$
β	= $\sqrt{1-M^2}$
$\Gamma_q(\eta_l),\Gamma_{qr}$	= spanwise loading function and coefficient in Eq. (13)
ξ,η	= coordinate variables of real wing
ξ',η'	= coordinate variables of imaginary wing
η_l	= η/s
θ	= transformed variable in F_2, Eq. (8), or angular spanwise parameter, Eq. (16)
θ_r	= $\pi r/(m+1)$
λ	= integer in spanwise integration of downwash
Λ	= $a(m+1)-1$
$\kappa_{r\lambda},\rho_{\nu r},\sigma_{\nu r},\tau_{\nu r}$	= coefficients in Eq. (24)
τ	= integration variable
τ_0	= $(1/\beta^2 r_l)(x_0-M\sqrt{x_0^2+r^2})$
ϕ	= angular chordwise parameter
ϕ_p	= $2\pi p/(2N+1)$
$\psi_q(\phi)$	= chordwise loading function in Eq. (13)
ω	= circular frequency of gust
$\bar{\omega}$	= $\omega/U\beta^2$
$\mathcal{M}\!\!\int$	= Manglar's principal value

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Introduction

IT is important to investigate the dynamic response to wind gusts of an airplane which flies with large angle of attack at the takeoff or landing condition where the airplane is easily destabilized by a slight disturbance. Needless to say, among all the parts of the airplane, the wing is most affected by the gust. Therefore, there have been many papers published on the gust response of wings since those of Sears.[1] However, we cannot find a paper on the gust response of a wing which is in a state of nonlinear condition, such as flying near the ground at large angle of attack. This nonlinearity, which appears in the response function (lift force), is a very complex phenomenon, and moreover, the distribution of the gust velocity might be nonuniform (shear flow) near the ground.

In this paper, an attempt is made to approach the gust response problem near the ground. However, the effect of shear flow is neglected, and moreover, the perturbation pressure field is related to the velocity potential by the ordinary differential equation as used in the linear theory. Accordingly, the lifting surface theory is used for the method of analysis. Furthermore, experiments were performed, and results are compared with calculated values.

An Approach by Lifting Surface Theory

Derivation of Kernel Function

Consider a wing which is flying in gusty air near the ground (altitude h) with freestream velocity U in x direction. This wing and its reference frame are shown in Fig. 1. Here, a thin wing and inviscid fluid flow are assumed. One of the familiar ways to express the existence of ground is by putting an image wing on the symmetrical point with regard to the ground. Then, two systems of coordinate axes 0-xyz and $0'$-$x'y'z'$ are taken on the real wing and on the image wing, respectively, as shown in Fig. 1. The phase of the image gust is inverted against the real gust.

Applying the lifting surface theory on this wing system, the following integral equation can be derived by the boundary condition on an arbitrary point $P(x,y,0)$ on the real wing:

$$\frac{\bar{w}(x,y,0)}{U} = -\frac{1}{8\pi}\lim_{z\to 0}\iint_S \bar{l}(\xi,\eta)K(x_0,y_0,z;M,\omega)\,d\xi d\eta$$

$$-\frac{1}{8\pi}\lim_{z\to 0}\iint_{S'} \bar{l}_I(\xi',\eta')K_I(x_0',y_0',z';M,\omega)\,d\xi' d\eta' \qquad (1)$$

where the double integration should be performed over the whole surface of real ($z=0$) and image ($z'=0$) wings, and subscript I indicates the value of the imaginary wing. Since the coordinate system of the real wing (0-xyz) is related with that of the image wing ($0'$-$x'y'z'$) as

$$x=x' \qquad y=y' \qquad z=z'-2h \qquad (2)$$

Therefore, Eq. (1) becomes

$$\frac{\bar{w}(x,y,0)}{U} = -\frac{1}{8\pi}\lim_{z\to 0}\iint_S \bar{l}(\xi,\eta)K(x_0,y_0,z;M,\omega)\,d\xi d\eta$$

$$-\frac{1}{8\pi}\lim_{z\to 0}\iint_S \bar{l}_I(\xi,\eta)K_I(x_0,y_0,z+2h;M,\omega)\,d\xi d\eta \qquad (3)$$

The kernel function in subsonic flow is given by the following well-known equation.[2,3]

$$K(x_0,y_0,z;M,\omega) = \frac{\partial^2}{\partial z^2}e^{-j\omega x_0/U}\int_0^{x_0}\frac{e^{j\frac{\omega}{U\beta^2}(\lambda - M\sqrt{\lambda^2+r^2})}}{\sqrt{\lambda^2+r^2}}\,d\lambda \qquad (4)$$

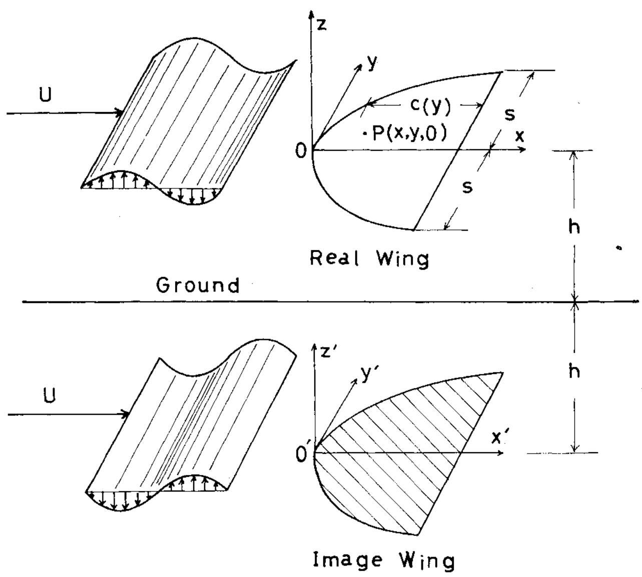

Fig. 1 Coordinate systems of a wing near the ground.

where

$$r=\beta\sqrt{y_0^2+z^2} \qquad \beta=\sqrt{1-M^2}$$

Relations between values of the real wing and the image wing can be derived by the boundary condition $\bar{w}=0$ at $h=0$ and Eq. (4) as follows:

$$K(x_0,y_0,-h;M,\omega) = K_I(x_0,y_0,h;M,\omega) \qquad (5a)$$

$$\bar{l}(\xi,\eta) = -\bar{l}_I(\xi,\eta) \qquad (5b)$$

In addition to the assumptions described in the foregoing section, it is assumed that the oscillation of the wing might be neglected. Then the downwash on the wing surface is replaced by the gust velocity as follows:

$$\bar{w}(x,y,0) = -\bar{w}_G e^{-j\omega x/U}$$

Therefore, the integral equation for the lift force of the wing is rearranged as

$$\frac{\bar{w}_G}{U}e^{-j\omega x/U} = \frac{1}{8\pi}\lim_{z\to 0}\left[\iint_S \bar{l}(\xi,\eta)\{K(x_0,y_0,z;M,\omega) - K(x_0,y_0,z+2h;M,\omega)\}\,d\xi d\eta\right] \qquad (6)$$

The generalized kernel function $K(x_0,y_0,z;M,\omega)$ was obtained in Refs. 4 and 5. However, in order to rearrange the kernel function applicable to the collocation method, the derivation is performed again with regard to Refs. 2 and 3. The kernel function is given by

$$K(x_0,y_0,z;M,\omega) = \frac{\partial^2}{\partial z^2}e^{-j\omega x_0/U}(F_1+F_2) \qquad (7)$$

where

$$F_1 = \int_0^\infty e^{-j\bar{\omega}\lambda}\frac{\exp(-j\bar{\omega}M\sqrt{\lambda^2+r^2})}{\sqrt{\lambda^2+r^2}}\,d\lambda$$

$$= K_0\left(\frac{\omega}{U}\sqrt{y_0^2+z^2}\right) - j\frac{\pi}{2}\left[I_0\left(\frac{\omega}{U}\sqrt{y_0^2+z^2}\right) - L_0\left(\frac{\omega}{U}\sqrt{y_0^2+z^2}\right)\right]$$

$$-\int_0^{M/\beta}\frac{\exp[-j(\omega/U\sqrt{y_0^2+z^2})\tau]}{\sqrt{1+\tau^2}}\,d\tau \qquad (8a)$$

$$F_2 = \int_0^{x_0} e^{j\bar{\omega}\lambda}\frac{\exp(-j\bar{\omega}M\sqrt{\lambda^2+r^2})}{\sqrt{\lambda^2+r^2}}\,d\lambda$$

$$= \int_0^{\sinh^{-1}(x_0/r)}\exp[j\bar{\omega}r\sinh\theta - M\cosh\theta)]\,d\theta \qquad (8b)$$

In F_2 the integration variable is transformed as

$$\lambda = r\sinh\theta$$

where

$$r=\beta\sqrt{y_0^2+z^2} \qquad \bar{\omega}=\omega/(U\beta^2)$$

After some calculations $\partial^2 F_1/\partial z^2$ and $\partial^2 F_2/\partial z^2$ can be obtained in the following forms:

$$\frac{\partial^2 F_1}{\partial z^2} = \frac{\omega}{U}\left(\frac{1}{r_1}-\frac{z^2}{r_1^3}\right)\Big\{-K_1\left(\frac{\omega}{U}r_1\right) - j\frac{\pi}{2}\Big[I_1\left(\frac{\omega}{U}r_1\right)$$

$$-L_1\left(\frac{\omega}{U}r_1\right)\Big]+\left(\frac{\omega z}{Ur_1}\right)^2\Big\{-K_1'\left(\frac{\omega}{U}r_1\right)$$

$$-j\frac{\pi}{2}\left[I_1'\left(\frac{\omega}{U}r_1\right)-L_1'\left(\frac{\omega}{U}r_1\right)\right]\Big\}$$

$$-\left(\frac{\omega}{U}\right)^2\left[1-\left(\frac{r}{r_1}\right)^2\right]\int_0^{M/\beta}\sqrt{1+\tau^2}e^{-j(\omega/U)r_1\tau}\mathrm{d}\tau$$

$$+\left(\frac{\omega}{U}\cdot\frac{z}{r_1}\right)^2\int_0^{M/\beta}\frac{\tau^2}{\sqrt{1+\tau^2}}e^{-j(\omega/U)r_1\tau}\mathrm{d}\tau$$

$$+j\frac{\omega}{U\beta r_1}\left[1-\left(\frac{z}{r_1}\right)^2\right]e^{-j(\omega/U\beta^2)Mr} \quad (9a)$$

$$\frac{\partial^2 F_2}{\partial z^2}=\left(\frac{\omega}{U}\cdot\frac{y_0}{r_1^2}\right)^2\int_0^{M/\beta}\sqrt{1+\tau^2}\,e^{-j(\omega/U)r_1\tau}\mathrm{d}\tau$$

$$+\left(\frac{\omega}{U}\cdot\frac{y_0}{r_1^2}\right)^2\int_0^{\tau_0}\sqrt{1+\tau^2}\,e^{j(\omega/U)r_1\tau}\mathrm{d}\tau$$

$$-\left(\frac{\omega}{U}\cdot\frac{z}{r_1^2}\right)^2\int_0^{M/\beta}\frac{\tau^2}{\sqrt{1+\tau^2}}e^{-j(\omega/U)r_1\tau}\mathrm{d}\tau$$

$$-\left(\frac{\omega}{U}\cdot\frac{z}{r_1^2}\right)^2\int_0^{\tau_0}\frac{\tau^2}{\sqrt{1+\tau^2}}e^{j(\omega/U)r_1\tau}\mathrm{d}\tau$$

$$-\Big[j\frac{\omega x_0}{U\beta^2\sqrt{x_0^2+r^2}}\left(\frac{z}{r^2}\right)^2(x_0-M\sqrt{x_0^2+r^2})$$

$$-j\frac{\omega}{U\beta^2}\left(\frac{y_0}{r_1^2}\right)(\sqrt{x_0^2+r^2}-Mx_0)$$

$$+\frac{1}{r_1^2}\frac{x_0}{\sqrt{x_0^2+r^2}}-\left(\frac{z}{r_1^2}\right)^2\frac{2x_0}{\sqrt{x_0^2+r^2}}$$

$$-\frac{\beta^2x_0}{(x_0^2+r^2)^{3/2}}\left(\frac{z}{r_1}\right)^2-j\frac{Mx_0\omega}{U(x_0^2+r^2)}\left(\frac{z}{r_1}\right)^2\Big]$$

$$\times\exp\left[j\frac{\omega}{U\beta^2}(x_0-\sqrt{x_0^2+r^2})\right]$$

$$-j\frac{\omega y_0^2}{U\beta}\cdot\frac{1}{r_1^3}e^{-j(\omega/U\beta)^2Mr} \quad (9b)$$

where

$$\tau_0=(1/r_1^2)(x_0-M\sqrt{x_0^2+r^2}) \qquad r_1=\sqrt{y_0^2+z^2}$$

and K_i's and I_i's are modified Bessel functions and L_i's are Struve functions.

Method of Solutions to the Integral Equation

Now, the integral equation [Eq. (6)] is rewritten as

$$\frac{\bar{w}_G}{U}=\frac{1}{8\pi}\int_{-s}^{s}\int_{x_l(\eta)}^{x_t(\eta)}\bar{l}(\xi,\eta)\left[\frac{1}{y_0^2}\bar{\bar{K}}(x_0,y_0,0;M,k)-\bar{K}(x_0,y_0,2h;M,k)\right]\mathrm{d}\xi\mathrm{d}\eta \quad (10)$$

where

$$\bar{\bar{K}}=e^{j(kx_0/d)}\frac{K}{|y_0|^2} \quad (11a)$$

$$\bar{K}=e^{j(kx_0/d)}K \quad (11b)$$

and $k(=\omega d/U)$ is reduced frequency. Represent the lift force distribution $\bar{l}$ as a series expansion of the form

$$\bar{l}(\xi,\eta_1)=\frac{8s}{\pi c(\eta_1)}\sum_{q=1}^{N}\Gamma_q(\eta_1)\psi_q(\phi) \quad (12)$$

where

$$\Gamma_q(\eta_1)=\frac{2}{m+1}\sum_{r=1}^{m}\left[\Gamma_q r\sum_{\mu=1}^{m}\sin\mu\theta\,\sin\mu\theta_r\right] \quad (13a)$$

$$\psi_q(\phi)=\frac{\cos(q-1)\phi+\cos q\phi}{\sin\phi} \quad (13b)$$

With these expressions Eq. (10) can be expressed as

$$\frac{\bar{w}_G}{U}=\frac{1}{2\pi}\int_{-1}^{1}\sum_{q=1}^{N}\Gamma_q(\eta_1)\left[\frac{1}{(y_1-\eta_1)^2}F_q-s^2F_{hq}\right]\mathrm{d}\eta_1 \quad (14)$$

where $y_1=y/s$, $\eta_1=\eta/s$ and

$$F_q=\frac{1}{\pi}\int_0^{\pi}\bar{\bar{K}}(x_0,y_0,0;M,k)\psi_q(\phi)\sin\phi\mathrm{d}\phi \quad (15a)$$

$$F_{hq}=\frac{1}{\pi}\int_0^{\pi}\bar{K}(x_0,y_0,2h;M,k)\psi_q(\phi)\sin\phi\,\mathrm{d}\phi \quad (15b)$$

Furthermore, θ_r and ϕ are new variables defined by

$$\eta_1=-\cos\theta \quad \eta_{1r}=-\cos\theta_r=-\pi r/(m+1) \quad (r=1,2,...,m) \quad (16a)$$

$$x=x_l(\eta_1)+[c(\eta_1)/2d](1-\cos\phi) \quad (16b)$$

Substituting Eqs. (13) and (15) into Eq. (14), this integral equation can finally be transformed into the simultaneous equation system

$$\frac{\bar{w}_G}{U}=\sum_{q=1}^{N}\sum_{r=1}^{m}\Gamma_{qr}\Omega_q(x,y_1;\eta_1) \quad (17)$$

where

$$\Omega_q(x,y_1;\eta_1)=\frac{1}{2\pi}\frac{2}{m+1}\sum_{\mu+1}^{N}\int_{-1}^{1}\sin\mu\theta\,\sin\mu\theta r$$

$$\times\Big\{\frac{R_q(x,y_1;\eta_1)}{\sin\theta}+\frac{1}{(\eta_1-y_1)^2}\Big[P_q(x,y_1;y_1)$$

$$+(\eta_1-y_1)P_q'(x,y_1;y_1)+\left(\frac{\beta s}{c(y_1)}\right)^2E_q(x,y_1)$$

$$\times\ln|y_1-\eta_1|\Big]\Big\}\mathrm{d}\eta_1 \quad (18)$$

In this equation, R_q is the regularized influence function defined by

$$R_q(x,y_1;\eta_1)=$$

$$\sin\theta\frac{P_q(x,y_1;\eta_1)-P_q(x,y_1;y_1)-(\eta_1-y_1)P_q'(x,y_1;y_1)}{(\eta_1-y_1)^2} \quad (19)$$

where

$$P_q(x,y_1;\eta_1)=F_q-s^2(\eta_1-y_1)^2F_{hq}$$

$$-\left(\frac{\beta s}{c(y_1)}\right)^2E_q(x,y_1)(y_1-\eta_1)^2\ln|y_1-\eta_1| \quad (20)$$

and $E_q(x,y_1)$ is the quantity for removal of logarithmic singularity. The concrete form of this quantity is obtained as $E_q(p,\nu)$ in the collocation process.[6]

Collocation Method

In order to solve the system of simultaneous equations [Eq. (17)], the collocation method is applied, i.e. the wing surface is divided into many small panels. Procedures are as follows[6]:

Division along the span direction:

$$y_{1_r} = -\cos\theta_r = -\cos\left(\frac{r\pi}{m+1}\right) \quad (r=1,2,...,m) \tag{21a}$$

$$\eta_{1_\lambda} = -\cos\theta_\lambda = -\cos\left(\frac{\lambda\pi}{\Lambda+1}\right) \quad (\lambda=1,2,...,\Lambda) \tag{21b}$$

where

$$\Lambda = \{a(m+1)-1\} \qquad (a=\text{positive number})$$

Division along the chord direction:

$$x_{p\nu} = x_l(y_{1_\nu}) + \tfrac{1}{2}c(y_{1_\nu})(1-\cos\phi_p)$$

$$= x_{l_\nu} + \tfrac{1}{2}c_\nu(1-\cos\phi_p) \tag{22a}$$

$$\phi_p = 2\pi p/(2N+1) \qquad (p=1,2,...,N) \tag{22b}$$

After these divisions, Ω_q [Eq. (18)] can be rewritten as follows:

$$\Omega_q(x_{p\nu},y_{1_\nu};\eta_{1r}) \stackrel{\Delta}{=} \Omega_q(p,\nu,r)$$

$$= \Big[\sum_{\lambda=1}^{\Lambda}\{R_q(p,\nu,\lambda)\kappa_{r\lambda}\} + P_q(p,\nu)\rho_{\nu r} + P'_q(p,\nu)\sigma_{\nu r} + (\beta^s/c_\nu)^2 E_q(p,\nu)\tau_{\nu r}\Big] \tag{23}$$

where

$$\kappa_{r\lambda} = \frac{(-1)^r \sin\theta_r \sin(\lambda\pi/a)}{2(m+1)(\Lambda+1)(\cos\theta_\lambda - \cos\theta_r)} \qquad (\theta_\lambda \neq \theta_r) \tag{24a}$$

$$= -1/2(\Lambda+1) \qquad (\theta_\lambda = \theta_r; \lambda = ar) \tag{24b}$$

$$\rho_{\nu r} = -\frac{\sin\theta_r\{1-(-1)^{r+\nu}\}}{2(m+1)(\cos\theta_\nu - \cos\theta_r)^2} \qquad (\theta_r \neq \theta_\nu) \tag{24c}$$

$$= (m+1)/4\sin\theta_\nu \qquad (\theta_r = \theta_\nu) \tag{24d}$$

$$\sigma_{\nu r} = -\frac{\sin\theta_r\{1-(-1)^{r+\nu}\}}{2(m+1)(\cos\theta_\nu - \cos\theta_r)} \qquad (\theta_r \neq \theta_\nu) \tag{24e}$$

$$= 0 \qquad (\theta_r = \theta_\nu) \tag{24f}$$

$$\tau_{\nu r} = -\frac{1}{m+1}\Big[\sin\theta_r(\tfrac{1}{4}\cos 2\theta_\nu - \tfrac{1}{2}\ln 2) - \sum_{\mu=2}^{m}\frac{\{\mu\sin\mu\theta_\nu\sin\theta_\nu + \cos\mu\theta_\nu\cos\theta_\nu\}\sin\mu\theta_r}{(\mu^2-1)}\tag{24g}$$

The foregoing form of $\kappa_{r\lambda}$ is proposed by Ichkawa,[7] who explains that no overlap occurs between integration points and control points by this expression.

Some Examples of Calculations and Comparison with Experimental Results

Results of Calculations

Calculations are performed for rectangular wings with three values of aspect ratio and one delta wing, which are used in experiments as described later. Mach number is taken as zero for all examples. The number of collocation points for the delta wing is illustrated in Fig. 2. The number of collocation points for the rectangular wings is the same as in the case of the delta wing. These collocation points are sufficient for convergence of results. This was clarified in the examples of Ref. 5. Numerical integrations are performed by Gauss' and Tchebycheff's methods. Furthermore, some modified Bessel functions and Struve functions are obtained by numerical integrations.

Lift force distributions of the rectangular wing ($\mathcal{R}=1.0$) and the delta wing are shown in Figs. 3 and 4, respectively. The local lift coefficient of the delta wing becomes infinitely large at the tip because the wing chord length is zero. In these

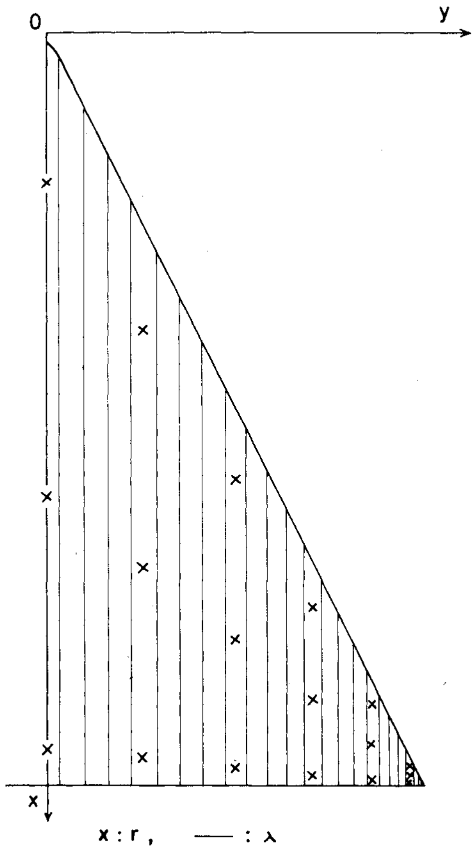

Fig. 2 Collocation points of the delta wing.

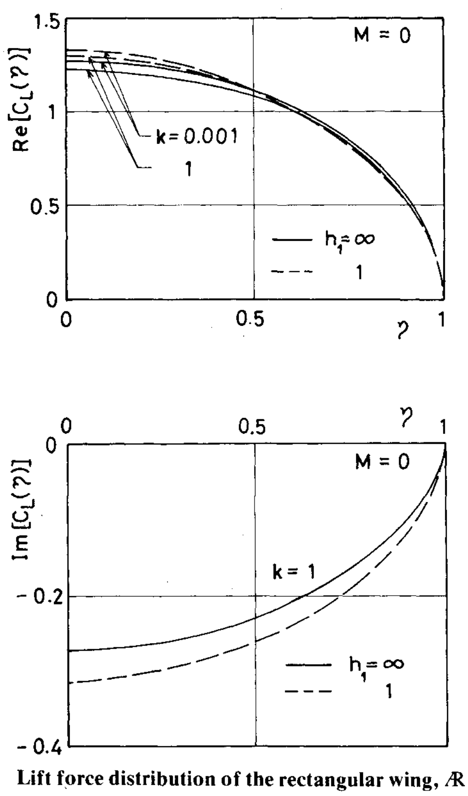

Fig. 3 Lift force distribution of the rectangular wing, $\mathcal{R}=1.0$.

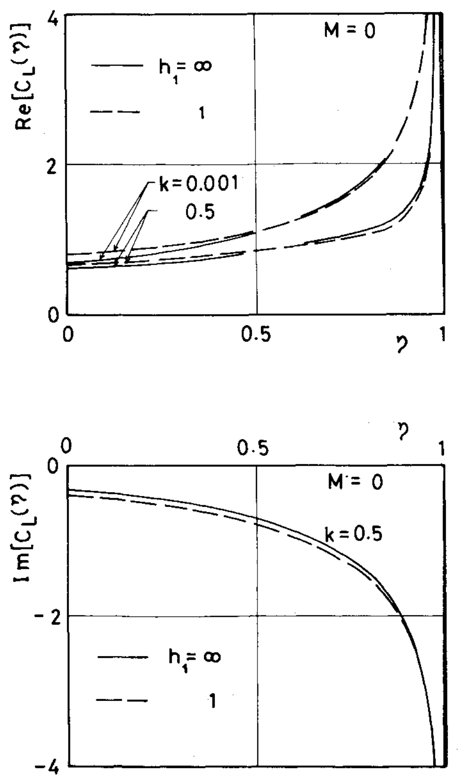

Fig. 4 Lift force distribution of the delta wing, Æ = 1.684.

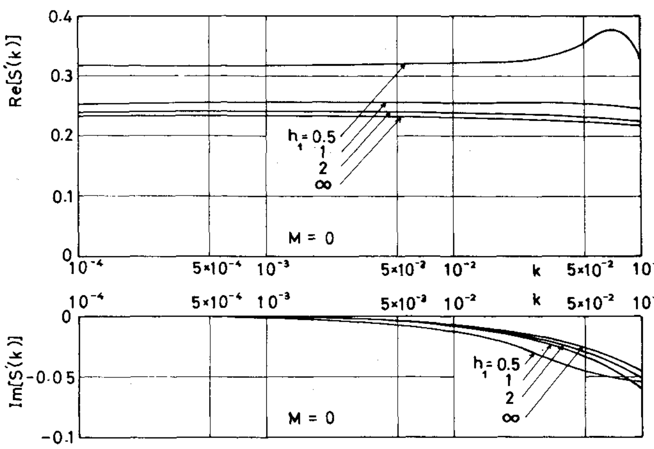

Fig. 5 Gust response function of the rectangular wing, Æ = 1.0.

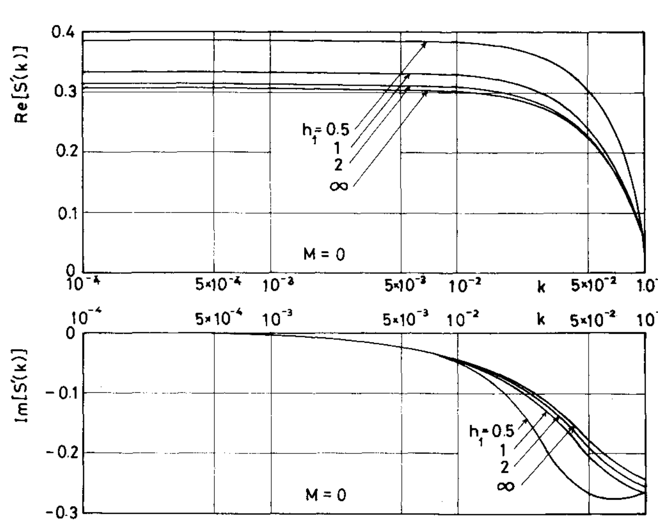

Fig. 6 Gust response function of the delta wing, Æ = 1.684.

figures the upper graph shows the real part, the lower graph shows the imaginary part, and altitude h is nondimensionalized by $\bar{c}$ (denoted by h_l). Figures 5 and 6 show the modified Sears' function $S'(k)$ which is defined by

$$S'(k) = \frac{1}{4\pi}\int\int_S e^{-jkx/\bar{c}}\,\bar{l}(x,y_l;M,k)\,dx\,dy_l \qquad (25)$$

in terms of the modified reduced frequency $k=\omega\bar{c}/U$. The parameter is the altitude of the wing, and the augmentation of the lift force by the ground effect is made clear by this calculation.

Comparison with Experimental Results

In order to examine the effect of the ground effect on the gust response of wings by experiments, we performed some wind-tunnel tests.[8] In these experiments, the unsteady lift force acting on the wing which is put in the gusty air flow of the wind tunnel is measured. The experimental apparatus is illustrated in Fig. 7. The normal gust was generated by oscillating cascades.

The experiments were performed for three rectangular wings of different aspect ratio and for one delta wing. The cross-sectional form of the rectangular wings is NACA-0012, and the aspect ratios are 1.0, 1.5, and 2.0. The chord length is 200 mm. The delta-wing model is illustrated in Fig. 8. These model wings were supported by two struts. The unsteady lift force was measured by a pair of strain gage balances which were set at the base of struts, and the gust was measured by x-type hot-wire anemometer. In Fig. 9 the variation of the gust is illustrated.

In order to compare experimental results with theoretical calculations, the augmentation of the lift force by the ground effect is obtained. This quantity is defined by $|S'(k)_h|/|S'(k)_\infty|$, i.e., the ratio of the absolute value of the modified Sears' function to that at $h=\infty$. Augmentation values of several conditions are illustrated in Figs. 10 and 11. Figure 10 shows the case of rectangular wings. It is found that the augmentation for the small aspect ratio wing is larger than that of the large aspect ratio wing, and also that the augmentation value is independent of k. Experimental values disperse to some extent. However, it can be said that both agree well with each other. The same fact is found in the delta wing case shown in Fig. 11. But in this case the augmentation decreases as k increases. This is caused by the pretty decrease of the imaginary part of $S'(k)$ for the delta wing. The comparison of the imaginary part of Fig. 6 with that of Fig. 5 makes this fact clear.

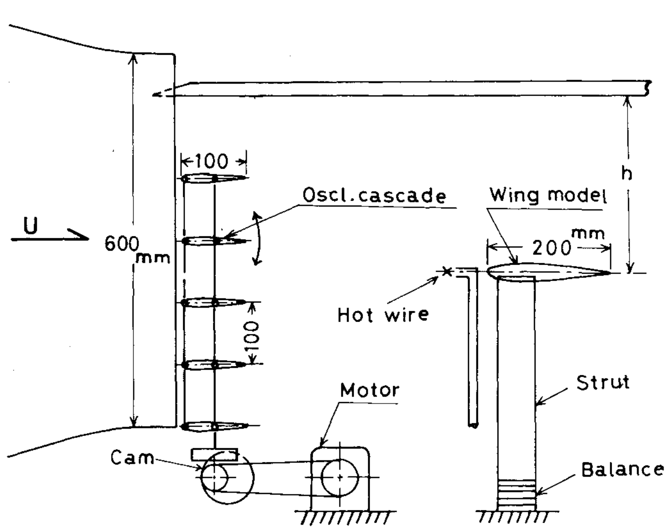

Fig. 7 Sketch of the experimental apparatus.

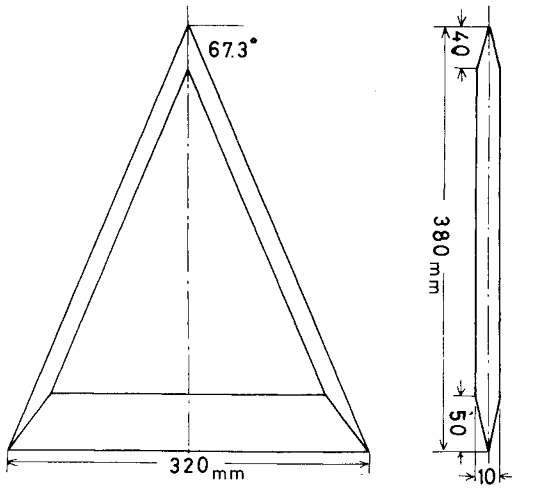

Fig. 8 Wind-tunnel model of the delta wing.

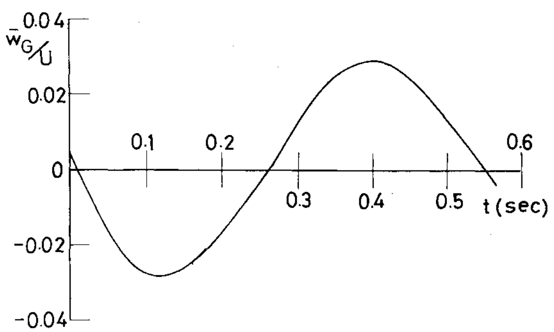

Fig. 9 Variation of the gust velocity.

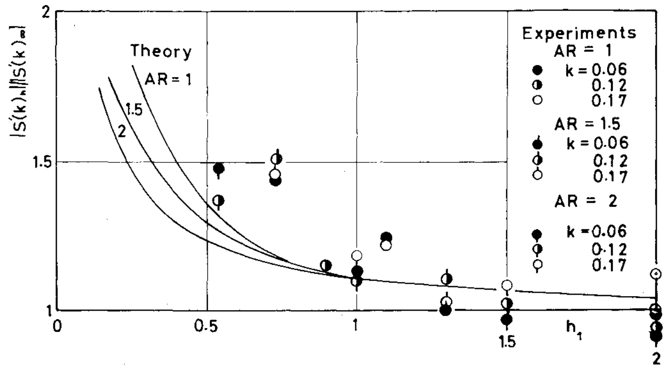

Fig. 10 Augmentation of the lift force of the rectangular wings.

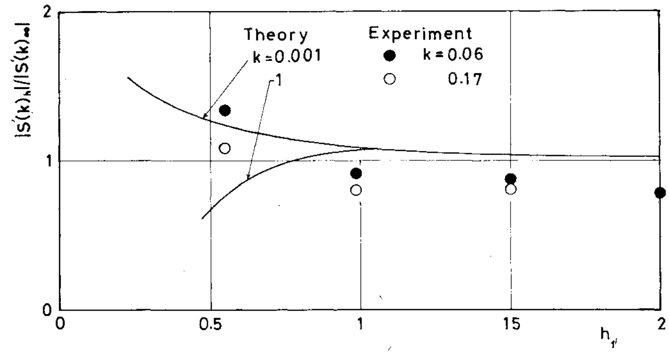

Fig. 11 Augmentation of the lift force of the delta wing, Æ = 1.684.

Conclusion

The gust response of wings which are flying near the ground is investigated. The lifting surface theory is used for theoretical calculations. The ground effect is described by the image-wing method. The generalized kernel function is obtained, and the numerical functions are calculated for the rectangular wings with aspect ratio 1.0, 1.5, and 2.0, and one delta wing. Furthermore, experiments of the ground effect in the gust response of wings are performed by wind-tunnel tests. Experimental values and theoretical results are compared by the augmentation of the lift force, and it is found that they agree well with each other.

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